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## Loanable Funds and Banking Optimization\*

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### Abstract

In this paper, we incorporate financial intermediaries' optimization into a model, and analyze the effect of a shock to banks' net worth on the economy with asymmetric information. In addition, we consider the effects of capital requirement regulations on the economy. Our main results are that a negative shock to banks' net worth negatively affects the macroeconomy and that to strengthen the capital requirement regulations leads to tighter bank lending. These results are consistent with prior research. And we have proved responses in interest rates (deposit rate and banks' loan rate) to change in banks' net worth.

**Keywords:** asymmetric information, banking optimization, net worth, capital requirements.

**JEL classification:** D82, G21

## 1 Introduction

Since the 1990s, the Japanese economy has been suffering from low growth and severe recessions. It has been often said in discussions of the recent Japanese economy that the financial sectors are responsible for these recessions. Almost all the banks in Japan hold many non-performing loans (NPLs). And they are engaged in writing off their NPLs and cannot afford to lend to borrowers. Since the Bank for International Settlements (BIS) regulations order banks to keep more than a certain level of capital ratios (capital divided by risk-weighted assets, mostly loans), banks which hold many NPLs have to tighten their loans in order to obey the regulations. Thus, banks' net worth plays an important role in lending. However, few studies has paid an attention

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to the specific effects of this factor on the economy. For this reason, we will discuss the role of banks' net worth on the economy in this paper.

This paper describes a theoretical study of how a change in financial intermediaries' net worth affects the economy. In particular, we investigate the role of the capital requirement regulations which financial intermediaries must obey in the overall economy with asymmetric information between lenders and borrowers.

According to Gale and Hellwig (1985), it is optimal, under asymmetric information, that the incentive-compatible debt contract between a lender and a borrower be a standard debt contract. Thus, we consider that lenders signed with borrowers in the form of debt contracts.

There are some studies that have considered the effects of asymmetric information on a macroeconomy. Stiglitz and Weiss (1981) consider an economy where firms with some net worth raise funds from intermediaries. They show that increasing interest rate could decrease the banks' profits. Holmstrom and Tirole (1997) study an economy where there are two types of lenders: households and financial intermediaries. They examine the roles of both banks' net worth and entrepreneurs' net worth on economy when both of them are subject to moral hazard problems. And they shows that reductions in both banks' net worth and entrepreneurs' net worth lead to less aggregate investment. Bernanke and Gertler (1989) analyze a "Financial Accelerator" mechanism, in which endogenous developments in credit market work to amplify and propagate shocks to the macroeconomy. In their paper, they conclude that it is borrowers' balance sheets that cause to macroeconomic fluctuations. In addition, Kiyotaki and Moore (1997) develop a model in which a change in the market price of a collateral is a major source of fluctuations.

All prior papers above except Holmstrom and Tirole (1997), however, had not analyzed the effects of banks' net worth on macroeconomy. Since we are interested in the role of banks' net worth, we need to develop a model where financial intermediaries have a certain net worth. Our model has similarities with Holmstrom and Tirole (1997), but differs from theirs in what financial intermediaries maximize. Repullo and Suarez (2000) also extend Holmstrom and Tirole's (1997) model, and analyze an effect of capital requirement regulations on economy. Their conclusions are that a fall in banks' net worth or strengthening capital requirement regulations will decrease bank lending and aggregate investment. But, our model is different from theirs because they

abstract from incentive problems at the level of banks. We consider the situation in which financial intermediaries lend their net worth with deposits from households to borrowers in order to maximize their profits.

The remainder of this paper is organized as follows. In Section 2 the optimal contracts between borrowers and lenders are considered. The model with financial intermediaries which have net worth is developed in Section 3. In Section 4, we study comparative statics, and the last section presents some conclusions.

## 2 The Optimal Contracts

Our economy consists of three types of agents: entrepreneurs, households and financial intermediaries. All agents are risk-neutral. Entrepreneurs can carry out projects. There are two types of projects: Project-1 and Project-2. It costs  $i$  ( $\leq 1$ ) to undertake a project. Entrepreneurs are endowed with some net worth  $\omega$ , and they are homogeneous except for their net worth. Net worth  $\omega$  is assumed to be distributed within the interval  $[0, 1]$  and to have a distribution function  $\Phi(\omega)$ , and a density function  $\phi(\omega)$ . Furthermore, if an entrepreneur whose net worth is less than  $i$  wants to carry out his project, he needs at least  $i - \omega$  in external funds. On the contrary, entrepreneurs with excess net worth will deposit their surplus net worth into financial intermediaries.

First, we consider projects. The (positive) returns and probabilities of success of their projects are described in Table 1.

We assume that

$$(1 + r)(i - \omega) < R_{g1}i < R_{g2}i, \quad (1)$$

$$p_1 R_{g1} + (1 - p_1)R_{b1} = p_2 R_{g2} + (1 - p_2)R_{b2}, \quad (2)$$

$$p_2 < p_1, \quad p_1 R_{g1} < p_2 R_{g2}, \quad (3)$$

where  $r$  is an interest rate of external funds.

Equation (1) implies that if an entrepreneur succeeds in his project, he can pay off his loan. Therefore (1) is a necessary condition for an entrepreneur to undertake a project. Equation (2) states that both projects have the same expected returns. And given Equation (3), we can get the ranking of returns:  $0 < R_{b2} < R_{b1} < R_{g1} < R_{g2}$ . From these conditions, we can conclude

that Project-1 is a low-risk project while Project-2 is high-risk.

An entrepreneur signs standard debt contracts on the basis of limited liability with lenders. Limited liability means that if a project is successful, then the borrower repays all his debt, but that if he fails in his project and he falls into insolvency, then he can declare default. Under the terms of a limited liability contract, the returns of a borrower and a lender depend on which project is undertaken.

We begin with considering the returns of borrowers. The returns of borrowers from Project-1 and Project-2 are, respectively, given by,

$$\begin{aligned} & p_1[R_{g1}i - (1+r)(i-\omega)] + (1-p_1) \max\{R_{b1}i - (1+r)(i-\omega), 0\} \\ & = \max\{p_1R_{g1}i + (1-p_1)R_{b1}i - (1+r)(i-\omega), p_1[R_{g1}i - (1+r)(i-\omega)]\}, \end{aligned} \quad (4)$$

$$\begin{aligned} & p_2[R_{g2}i - (1+r)(i-\omega)] + (1-p_2) \max\{R_{b2}i - (1+r)(i-\omega), 0\} \\ & = \max\{p_2R_{g2}i + (1-p_2)R_{b2}i - (1+r)(i-\omega), p_2[R_{g2}i - (1+r)(i-\omega)]\}. \end{aligned} \quad (5)$$

From Equations (2) and (3), it is assured that the return on Project-2 is not less than the return from Project-1. Therefore we can conclude that borrowers prefer Project-2 (high-risk project) to Project-1 (low-risk project).

Next, we examine the returns lenders receive. The returns of lenders are, respectively,

$$\begin{aligned} & p_1(1+r)(i-\omega) + (1-p_1) \min\{(1+r)(i-\omega), R_{b1}i\} \\ & = \min\{(1+r)(i-\omega), p_1(1+r)(i-\omega) + (1-p_1)R_{b1}i\}, \end{aligned} \quad (6)$$

$$\begin{aligned} & p_2(1+r)(i-\omega) + (1-p_2) \min\{(1+r)(i-\omega), R_{b2}i\} \\ & = \min\{(1+r)(i-\omega), p_2(1+r)(i-\omega) + (1-p_2)R_{b2}i\}. \end{aligned} \quad (7)$$

Under Equations (2) and (3), it follows that the return on Project-1 is not less than that from Project-2. Hence we can conclude that lenders prefer Project-1 (low-risk project) to Project-2 (high-risk project).

The financial sector consists of a lot of financial intermediaries. Therefore, it is fair to say that the financial sector is competitive. Here, we normalize the number of financial intermediaries to unity. We suppose that financial intermediaries monitor their borrowers with some monitoring costs so as to prevent borrowers from undertaking high-risk projects. And we suppose that financial intermediaries have their own net worth.

Finally, we consider households. Households are inferior to financial intermediaries with regard to the ability to collect information, and hence they cannot monitor their borrowers' actions. It is assumed that households can deposit their assets in financial intermediaries. Deposits are paid back to households in the form of interest and principal payments. Under the assumption of a competitive financial market, an interest rate of direct finance  $r_d$  is equal to a deposit rate  $\bar{r}$ . Therefore, households are indifferent about whether they lend their assets directly to borrowers or deposit in intermediaries.

## 2.1 The Optimal Contracts: The Case of Direct Finance

We begin with examining direct finance. Since lenders cannot monitor their borrowers in the case of direct finance, borrowers may carry out high-risk projects (Project-2).

From the fact that borrowers who raise funds for their projects directly from households prefer Project-2, the returns of borrowers (entrepreneurs) and lenders (households) are given by, respectively,

$$p_2[R_{g2}i - (1 + \bar{r})(i - \omega)] + (1 - p_2) \max\{R_{b2}i - (1 + \bar{r})(i - \omega), 0\}, \quad (8)$$

$$p_2(1 + \bar{r})(i - \omega) + (1 - p_2) \min\{(1 + \bar{r})(i - \omega), R_{b2}i\}. \quad (9)$$

If (9) is less than those from deposits, lenders may deposit their assets in financial intermediaries instead of lending. Hence, we can find a necessary condition for direct finance as,

$$p_2(1 + \bar{r})(i - \omega) + (1 - p_2)R_{b2}i \geq (1 + \bar{r})(i - \omega). \quad (10)$$

Let  $\bar{\omega}(\bar{r})$  be the minimum level of borrowers' net worth to raise funds through direct finance. Then,  $\bar{\omega}(\bar{r})$  is defined as follows,

$$\begin{aligned} p_2(1 + \bar{r})(i - \bar{\omega}(\bar{r})) + (1 - p_2)R_{b2}i &= (1 + \bar{r})(i - \bar{\omega}(\bar{r})) \\ \bar{\omega}(\bar{r}) &= i \left[ 1 - \frac{R_{b2}}{1 + \bar{r}} \right]. \end{aligned} \quad (11)$$

No entrepreneurs with  $\omega$  less than  $\bar{\omega}(\bar{r})$  can raise funds through direct finance, because the returns of lenders from lending to entrepreneurs with  $\omega(< \bar{\omega}(\bar{r}))$  are less than those from deposits. In this case, lenders deposit their assets into financial intermediaries instead. For the reasons stated above, entrepreneurs must have their net worth  $\omega$  over  $\bar{\omega}(\bar{r})$  in order to raise funds

through direct finance. Since entrepreneurs whose net worth are smaller than  $i$  need  $i - \omega$  in external funds to undertake their projects, we can show that the total of external funds through direct finance is

$$L_d(\bar{r}) = \int_{\bar{\omega}(\bar{r})}^i (i - \omega) d\Phi(\omega). \quad (12)$$

## 2.2 The Optimal Contracts: The Case of Indirect Finance

Next, we consider indirect finance. In the case of indirect finance, lenders can monitor their borrowers, so borrowers undertake low-risk projects (Project-1). Suppose no lenders can monitor their borrowers without any monitoring cost.

Allowing for the monitoring cost, indirect-finance contracts are signed. Hence, the indirect-finance loan rate  $r_i$  is higher than the direct-finance loan rate  $\bar{r}$ . Here we think that financial intermediaries can get return  $\gamma$  if they monitor their borrowers by the monitoring intensity  $\beta$ . And we assume that  $\gamma = F(\beta)$ , where  $F' > 0$ ,  $F'' < 0$ . (See Figure 1.)

We solve the function  $F$  for  $\beta$ , and we get  $\beta = F^{-1}(\gamma) = \tau(\gamma)$ , where  $\tau' > 0$ ,  $\tau'' > 0$ . We suppose that  $r_i = \bar{r} + \gamma$ . In addition, we assume that if households deposit their assets into financial intermediaries, they can get interest and principal payments at the deposit rate  $\bar{r}$ .

Let  $n$  be the amount of a financial intermediary's net worth which he lends to a borrower. The returns of borrowers, financial intermediaries and households are written as follows,

$$p_1[R_{g1}i - (1 + \bar{r} + \gamma)(i - \omega)] + (1 - p_1) \max\{R_{b1}i - (1 + r_i)(i - \omega), 0\}, \quad (13)$$

$$p_1(1 + \bar{r} + \gamma)(i - \omega) + (1 - p_1) \min\{(1 + \bar{r} + \gamma)(i - \omega), R_{b1}i\} \\ - (1 + \bar{r})(i - n - \omega) - \tau(\gamma)(i - \omega), \quad (14)$$

$$(1 + \bar{r})(i - n - \omega), \quad (15)$$

where  $\gamma > \tau(\gamma) = A\gamma^\alpha$ , ( $\alpha > 1$ ).

While the returns of households are independent of whether borrowers are insolvent or not, the returns of entrepreneurs and financial intermediaries depend on the results of projects. In case of insolvency, the returns of financial intermediaries are,

$$R_{b1}i - (1 + \bar{r})(i - n - \omega) - \tau(\gamma)(i - \omega). \quad (16)$$

Therefore, we obtain the following necessary condition for indirect finance,

$$p_1(1 + \bar{r} + \gamma)(i - \omega) + (1 - p_1)R_{b1}i - (1 + \bar{r})(i - n - \omega) - \tau(\gamma)(i - \omega) \geq (1 + \bar{r})n. \quad (17)$$

Equation (17) means that the returns from lending to entrepreneurs must be no less than the opportunity cost.<sup>1</sup> Now we find the minimum level of borrowers' net worth  $\underline{\omega}(\bar{r}, \gamma)$  in order to raise external funds using indirect finance in the same way as in the case of direct finance,

$$p_1(1 + \bar{r} + \gamma)(i - \underline{\omega}) + (1 - p_1)R_{b1}i - (1 + \bar{r})(i - n - \underline{\omega}) - \tau(\gamma)(i - \underline{\omega}) = (1 + \bar{r})n$$

$$\underline{\omega}(\bar{r}, \gamma) = i \left[ 1 - \frac{(p_1 - 1)R_{b1}}{p_1(1 + \bar{r} + \gamma) - (1 + \bar{r} + \tau(\gamma))} \right]. \quad (18)$$

From these observations, we conclude that entrepreneurs must have their net worth  $\omega$  over  $\underline{\omega}$  to raise funds through indirect finance. From the fact that indirect finance is more costly by the monitoring cost than direct finance, we derive the following condition which the monitoring cost  $\gamma$  must satisfy,<sup>2</sup>

$$\bar{\omega}(\bar{r}) \equiv i \left[ 1 - \frac{R_{b2}}{1 + \bar{r}} \right] > i \left[ 1 - \frac{(p_1 - 1)R_{b1}}{p_1(1 + \bar{r} + \gamma) - (1 + \bar{r} + \tau(\gamma))} \right] \equiv \underline{\omega}(\bar{r}, \gamma)$$

$$\tau(\gamma) - p_1\gamma > \left[ \frac{R_{b1}}{R_{b2}} - 1 \right] (1 - p_1)(1 + \bar{r}). \quad (19)$$

Under this condition, we conclude that the aggregate quantities of indirect lending are,

$$L_i(\bar{r}, \gamma) = \int_{\underline{\omega}(\bar{r}, \gamma)}^{\bar{\omega}(\bar{r})} (i - \omega) d\Phi(\omega). \quad (20)$$

### 3 The Banking Optimization

Now, we are ready to analyze our model. The economy contains a continuum of agents and infinitely lived financial intermediaries. Each agent is endowed with some net worth  $\Omega$ .  $\Omega$  is assumed to be uniformly distributed. All agents must provide a given  $N$  from their endowments for financial intermediaries. Since we suppose that a financial intermediary lends  $n$  to an entrepreneur from his own net worth, we can state that the relation between  $N$  and  $n$  as follows,

$$N = n[\Phi(\bar{\omega}(\bar{r})) - \Phi(\underline{\omega}(\bar{r}, \gamma))]. \quad (21)$$

<sup>1</sup>We regard the returns from deposits as opportunity costs.

<sup>2</sup>If this condition is not satisfied, then no entrepreneurs may raise funds by means of indirect finance.



At the end of the period, they sell their shares  $N$  and dividends  $\pi$  from their ownership of financial intermediaries. Since the agents have no bequest motive, they consume all of their assets.

Here, let  $\omega \equiv \Omega - N$  denote the amount of net worth per capita. Then, since endowments  $\Omega$  are uniformly-distributed, a distribution function of  $\omega$ ,  $\Phi(\omega)$ , also becomes uniform from the logarithmic utility function. We normalize the population to unity, and assume that  $\omega$  is uniformly distributed between  $[0, 1]$ . Under these assumptions, we can divide the agents into three types according to their net worth carried over to the next period.

1. Agents who can raise external funds through direct finance:  $\omega \geq \bar{\omega}$ .
2. Agents who can raise external funds using indirect finance:  $\underline{\omega} \leq \omega < \bar{\omega}$ .
3. Agents who cannot finance:  $\omega < \underline{\omega}$ .

While we call agents 1 and 2 entrepreneurs, agent 3 is called households. We can show the interaction among households, entrepreneurs, and financial intermediaries as in Figure 2.

Here, we analyze the equilibrium in the credit market. Financial intermediaries inelastically receive households' assets as deposits, and unite them with intermediaries' net worth  $N$  and lend to entrepreneurs. We can write the clearing condition of credit markets as follows,

$$\int_{\underline{\omega}(\bar{r}, \gamma)}^i (i - \omega) d\omega = N + \int_0^{\underline{\omega}(\bar{r}, \gamma)} \omega d\omega + \int_i^1 (\omega - i) d\omega. \quad (22)$$

LHS of (22) is the demand for external funds. And RHS of (22) is the supply of credit (intermediaries' net worth plus households' endowment and entrepreneurs' excess net worth).

Next, we consider banking action in this economy. Financial intermediaries lend their net worth and deposits to lenders. Suppose that a financial authority sets  $\eta$  as a net worth ratio that all financial intermediaries must hold. Hence, a representative financial intermediary maximizes his profit with respect to  $\gamma$ ,

$$\begin{aligned} \max_{\gamma} \Pi = & \int_{\bar{\omega}}^{\bar{\omega}} [(1 + \bar{r} + \gamma)(i - \omega) - (1 + \bar{r})(i - n - \omega) - \tau(\gamma)(i - \omega)] d\omega \\ & + \int_{\underline{\omega}}^{\bar{\omega}} [p_1(1 + \bar{r} + \gamma)(i - \omega) + (1 - p_1)R_{b1}i - (1 + \bar{r})(i - n - \omega) - \tau(\gamma)(i - \omega)] d\omega, \end{aligned} \quad (23)$$

$$s.t. \quad N \geq \eta L_i(\bar{r}, \gamma). \quad (24)$$

where  $\hat{\omega} = i[1 - \frac{R_{b1}}{1+\bar{r}+\gamma}]$ , the threshold that an entrepreneur can repay his obligation even when his project is unsuccessful. From the fact  $N = n(\bar{\omega} - \underline{\omega})$ , we obtain the following first-order condition,

$$\begin{aligned} & \int_{\hat{\omega}(\bar{r}, \gamma)}^{\bar{\omega}(\bar{r})} (1 - \tau'(\gamma))(i - \omega) d\omega - [(\gamma - \tau(\gamma))(i - \hat{\omega}) + n(1 + \bar{r})] \frac{\partial \hat{\omega}(\bar{r}, \gamma)}{\partial \gamma} \\ & + \int_{\underline{\omega}(\bar{r}, \gamma)}^{\hat{\omega}(\bar{r}, \gamma)} (p_1 - \tau'(\gamma))(i - \omega) d\omega \\ & + [\{(p_1 - 1)(1 + \bar{r}) + p_1\gamma - \tau(\gamma)\}(i - \hat{\omega}) + (1 - p_1)R_{b1}i + n(1 + \bar{r})] \frac{\partial \hat{\omega}(\bar{r}, \gamma)}{\partial \gamma} \\ & - [\{(p_1 - 1)(1 + \bar{r}) + p_1\gamma - \tau(\gamma)\}(i - \underline{\omega}) + (1 - p_1)R_{b1}i + n(1 + \bar{r})] \frac{\partial \underline{\omega}(\bar{r}, \gamma)}{\partial \gamma} \\ & + \lambda \eta(i - \underline{\omega}) \frac{\partial \underline{\omega}(\bar{r}, \gamma)}{\partial \gamma} = 0, \end{aligned} \quad (25)$$

$$N - \eta(\bar{\omega}(\bar{r}) - \underline{\omega}(\bar{r}, \gamma)) \left[ i - \frac{1}{2}(\bar{\omega}(\bar{r}) + \underline{\omega}(\bar{r}, \gamma)) \right] \geq 0, \quad \lambda \geq 0. \quad (26)$$

First, we study the case where the capital requirement regulation is not bound. This may be the case where financial intermediaries have relatively plentiful net worth to their lendings. In this case, the capital requirement regulation (24) is not bound. Hence,  $\lambda$  must be zero.

We can rewrite the above first-order condition (26) as follows,

$$N - \eta(\bar{\omega}(\bar{r}) - \underline{\omega}(\bar{r}, \gamma)) \left[ i - \frac{1}{2}(\bar{\omega}(\bar{r}) + \underline{\omega}(\bar{r}, \gamma)) \right] > 0, \quad \lambda = 0.$$

Then, we get the optimal  $\bar{r}^*$  and  $\gamma^*$ . But, we cannot find explicit solutions. So, we solve these equations implicitly, and we get,<sup>3</sup>

$$\bar{r}^* = \bar{r}(N), \quad \frac{\partial \bar{r}^*}{\partial N} < 0, \quad (27)$$

$$\gamma^* = \gamma(N), \quad \frac{\partial \gamma^*}{\partial N} < 0. \quad (28)$$

By Equations (27) and (28), we can get the optimal indirect-finance rate  $r_i^*$  straightforwardly,

$$r_i^* = \bar{r}^* + \gamma^* = r_i(N), \quad \frac{\partial r_i^*}{\partial N} < 0. \quad (29)$$

If banks' net worth increases, the supply of credit must also increase. This increase leads to a lower deposit rate and a lower bank loan rate. These results are consistent with our intuition.

<sup>3</sup>Refer to Appendix for a detailed calculation.

Next, we investigate the case where the capital requirement regulation is bound. From Equations (25), (26), and (22), we get the optimal  $\bar{r}^{**}$ ,  $\gamma^{**}$  and  $\lambda^{**}$  implicitly.  $\bar{r}^{**}$  and  $\gamma^{**}$  is written as follows,<sup>4</sup>

$$\bar{r}^{**} = \bar{r}(N, \eta), \quad \frac{\partial \bar{r}^{**}}{\partial N} > 0, \quad \frac{\partial \bar{r}^{**}}{\partial \eta} < 0, \quad (30)$$

$$\gamma^{**} = \gamma(N, \eta), \quad \frac{\partial \gamma^{**}}{\partial N} < 0, \quad \frac{\partial \gamma^{**}}{\partial \eta} > 0. \quad (31)$$

Using the results above, we can discuss the effects of  $N$  and  $\eta$  in the optimal indirect-finance loan rate  $r_i^{**}$ .

$$r_i^{**} = \bar{r}^{**} + \gamma^{**} = r_i(N, \eta), \quad \frac{\partial r_i^{**}}{\partial N} < 0, \quad \frac{\partial r_i^{**}}{\partial \eta} > 0. \quad (32)$$

## 4 Comparative Statics

Our main interest is in the effects that changes in banks' net worth and capital requirement ratio have on the bank lendings. For this reason, we investigate the effect of the changes in  $N$  and  $\eta$ .

First, we study the effect of negative shocks on financial intermediaries' net worth or decrease in  $N$  when the capital requirement regulation is not bound. In this case, both the minimum level of net worth  $\bar{\omega}^*$  and  $\underline{\omega}^*$ , which entrepreneurs who raise external funds from direct finance and indirect finance must have respectively, increase. Since  $\frac{d\bar{\omega}^*}{dN}$  is larger than  $\frac{d\underline{\omega}^*}{dN}$ , we can conclude that the total number of entrepreneurs who are financed by banks decreases.

Next, we discuss the same effect when the capital requirement regulation is bound. We may say that this is the case of Resona Bank which has been denied provision for deferred tax assets by the auditing company and where capital requirement regulation is binding. From equations (30) and (32), and the fact that  $|\frac{\partial \bar{r}^{**}}{\partial N}| < |\frac{\partial \gamma^{**}}{\partial N}|$ , when  $N$  decreases, while the deposit rate  $\bar{r}^{**}$  falls, the indirect-finance loan rate  $r_i^{**}$  rises. This means that the minimum level of borrowers' net worth  $\bar{\omega}^{**}$  decreases and  $\underline{\omega}^{**}$  increases. As a result, the total number of entrepreneurs who raise funds through indirect finance ( $\underline{\omega}^{**} \leq \omega < \bar{\omega}^{**}$ ) decreases. One of the important problems in the Japanese economy is the problem of NPLs. According to this result, the occurrence of new NPLs leads to a shortage of banks' net worth and to tighter bank lending.

<sup>4</sup>In this case, we can get the optimal solution explicitly. But we omit the detailed calculation.

And finally, we investigate the effect of a rise in the capital requirement ratio. When a financial authority raises the capital requirement ratio, the deposit rate  $r^{**}$  decreases, but the return for monitoring  $\gamma^{**}$  and indirect-finance loan rate  $r_i^{**}$  increase. And we get that while the minimum level of borrowers' net worth in order to raise funds through direct finance  $\bar{\omega}^{**}$  declines, a counterpart in order to raise funds using indirect finance  $\underline{\omega}^{**}$  is constant. This is because a rise in return for monitoring is offset by a decline in deposit rate. We conclude that the total number of entrepreneurs has not changed, but since some entrepreneurs who raise external funds through indirect finance shift to direct finance, the aggregate of bank lending decreases.

Table 2 summarizes a effect of the rise in banks' net worth  $N$  and the capital requirement ratio  $\eta$ . The results of our analysis are summarized in:

**Proposition 1.** *Both a decrease in the banks' net worth  $N$  and an increase in the capital requirement regulation  $\eta$  lead to tighter the banks' lendings. And a decrease in the bank's net worth  $N$  decrease the total number of entrepreneurs.*

This proposition is consistent with prior research. Although Repullo and Suarez (2000) are different from ours because they abstract from banking optimization, their results and ours are the same.

## 5 Concluding Remarks

We have studied some effects of asymmetric information on the macroeconomic fluctuations in this paper. Our model basically follows Holmstrom and Tirole's (1997) model. However, it has differences in two respects. First, our model differs from theirs with respect to the returns and probabilities of success of projects. Second, and more importantly, we have considered the financial intermediaries' optimization explicitly. Our results, however, are consistent with several previous papers. (e.g., Holmstrom and Tirole, 1997, Repullo and Suarez, 2000).

The agents are endogenously divided into three types according to their net worth: two types of entrepreneurs and households. While entrepreneurs who can raise external funds through direct finance undertake high-risk projects, those who can raise funds using indirect finance carry out low-risk projects because of monitoring by financial intermediaries.

The model shows that the effects of negative shocks on banks' net worth and the strengthening of capital requirement regulations lead to tighter bank lendings. This means that it becomes more difficult for small and less-capitalized firms to get bank loans. We think that tighter bank lendings in the Japanese economy have their roots in the shortage of banks' net worth. Therefore, if a government wants to prevent banks from tightening their lendings, the government should inject capital into banks which have become less capitalized. According to this model, we think it is reasonable for the Japanese government to support some major banks and inject public funds into them.

In addition, where the capital requirement regulation is binding, we find that when banks' net worth contracts, deposit interest rate  $\bar{r}$  falls, whereas indirect finance rate  $r_i$  rises. Recently, it is said that some Japanese major banks have lapsed into shortage of net worth. And, while the demand deposit rate is very low (about 0.001%), the bank loan rate is still high (2–3%) or tends to rise. As a result, many less-capitalized small firms can not raise their funds. We think that this result has agreed with the actual Japanese economy very much. This result seems to be as the same as that of Holmstrom and Tilore (1997) in case of variable investment scale. In case of fixed investment scale, however, they have showed that at least one of the interest rates,  $\bar{r}$  or  $r_i$ , must go up when there is a capital squeeze. That is, it is ambiguous which of the interest rates goes up. We have solved this ambiguity by showing that deposit rate  $\bar{r}$  decreases and loan rate  $r_i$  rises when banks' net worth contracts.

## Appendix

In this appendix, we prove the existence of solutions in the case where capital requirement regulation is not bound.

First, substitute  $\lambda = 0$  into Equation (25), and rearrange it as follows,

$$(1-p_1) \left( \frac{R_{b1}}{1+\bar{r}+\gamma} \right)^2 + (\tau'(\gamma)-1) \left( \frac{R_{b2}}{1+\bar{r}} \right)^2 + (p_1-\tau'(\gamma)) \left[ \frac{(p_1-1)R_{b1}}{p_1(1+\bar{r}+\gamma) - (1+\bar{r}+\tau(\gamma))} \right]^2 = 0.$$

Then solve Equation (22) for  $\bar{r}$ , and substitute it into the equation above. Since the first term is always positive, either the second term or the third term must be negative.

Here, let  $G(\gamma)$  be the LHS of the equation above. Define  $\gamma_1$  and  $\gamma_2$  the value of  $\gamma$  such that  $\tau'(\gamma) = p_1$  and  $\tau'(\gamma) = 1$ , respectively. We find that  $G(\gamma_1)$  is positive whereas  $G(\gamma_2)$  becomes

negative. From these results and the fact that  $G$  is a continuous and decreasing function in  $\gamma \in (\gamma_1, \gamma_2)$ , if the optimal  $\gamma^*$  satisfied second-order condition,  $G'(\gamma^*) < 0$ , we can get a unique solution  $\gamma^*$  between  $\gamma_1 < \gamma^* < \gamma_2$ . Moreover, we can find that a rise in  $N$  shifts  $G(\gamma)$  downwardly. This allows us to conclude that the optimal  $\gamma^*$  is decreasing in  $N$ .

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Project	State	Probability	Return
1	Good	$p_1$	$R_{g1}$
	Bad	$1 - p_1$	$R_{b1}$
2	Good	$p_2$	$R_{g2}$
	Bad	$1 - p_2$	$R_{b2}$

Table 1: Returns and probabilities of success of each project

	$\bar{r}^*$	$\gamma^*$	$r_i^*$	$\bar{\omega}^*$	$\underline{\omega}^*$	$\bar{\omega}^* - \underline{\omega}^*$
$N$	—	—	—	—	—	+
	$\bar{r}^{**}$	$\gamma^{**}$	$r_i^{**}$	$\bar{\omega}^{**}$	$\underline{\omega}^{**}$	$\bar{\omega}^{**} - \underline{\omega}^{**}$
$N$	+	—	—	+	—	+
$\eta$	—	+	+	—	0	—

Table 2: The effects of the change in  $N$  and  $\eta$

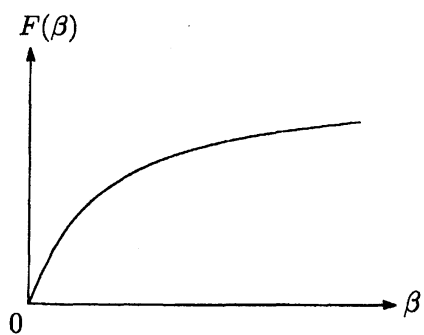


Figure 1: The relationship between the return and the monitoring intensity

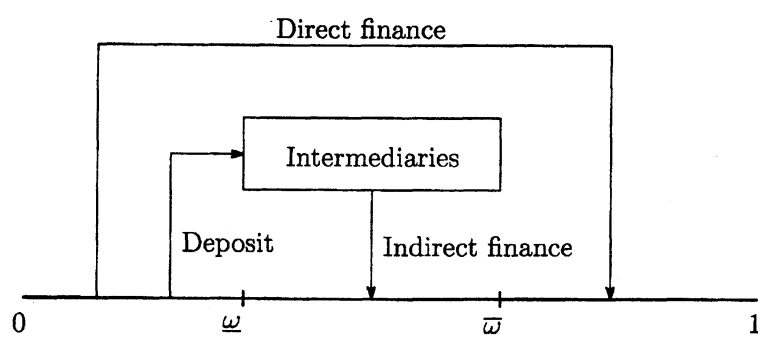


Figure 2: The relationship between agents